AIAA JOURNAL
Vol. 42, No. 8, August 2004

Successive Matrix Inversion Method for Reanalysis of Engineering Structural Systems

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Over the past several decades, numerous structural analysis techniques have been developed to represent physical systems behavior more realistically, and the structural models, therefore, have become larger and more complex. Even though modern computer power has increased significantly, cost of computational analysis has been a major restrictive factor in a structural system design involving multiple disciplines and repetitive simulations. A new reanalysis technique is developed. The successive matrix inversion (SMI) method is most suitable for reanalysis of structures. The SMI method reproduces exact solutions for any localized modification of the initial system. Several numerical examples are given to demonstrate the efficiency of this method.

Nomenclature

A	=	cross-sectional area of members
b	=	width of members
$\{d_0\}$, $\{d\}$	=	original and modified displacement vectors
E	=	Young's modulus
$\{f\}$	=	load vector
h	=	height of members
$[I]$	=	identity matrix
$[K_0]$, $[K]$	=	original and modified global stiffness matrices
N	=	total number of degrees of freedom
n	=	total number of columns that have nonzero elements in $[\Delta K]$
t	=	thickness of members
$\{t_0\}$,$\{\Delta t\}$	=	thickness vectors of the original and the modification
x, y, z	=	Cartesian (global) coordinate system
$[\Delta K]$	=	stiffness modification matrix

Superscript

T	=	transposition

I. Introduction

As the size and complexity of a computational model for an engineering system increases, it is strongly desired to incorporate an accurate and inexpensive reanalysis methodology in the fields of structural analysis, such as system behavior prediction, structural optimization, structural reliability, and so forth. Motivation for reanalysis is to regenerate the modified results using the previous simulation information with much less cost compared to the complete additional simulations of the modified systems.

Received 20 August 2003; revision received 5 February 2004; accepted for publication 16 February 2004.

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One of the popular reanalysis methods is function approximation, usually based on series expansion[1–4] or design of experiments.[5–7] Generally, a function approximation method constructs closed-form equations for an implicit analysis, such as finite element analysis (FEA), in terms of system design variables for the responses of a target system, including displacements, fundamental frequency, internal stresses, and so on. Once the approximate model is obtained, the responses of the system can be regenerated without the actual FEA simulations. The cost for the reanalysis using function approximation techniques is trivial because the solutions are calculated from closed-form equations in place of FEA simulations. However, the approximate model solutions are accurate only within specific bounds, which depend on the efficacy or accuracy of a specific approximation method and the characteristics of the target system.

Other reanalysis methods are based directly on the matrix equilibrium equations in the stiffness method. These methods include the direct method,[8–10] iterative method,[11] combined method,[12] and so forth. In the direct method based on the Sherman–Morrison identity,[8] the main idea is to obtain system responses without computing an inversion of the modified stiffness matrix in a FEA formulation. The Sherman–Morrison formula gives an exact solution for a modified system. However, the modification of the system matrix is decomposed into two vectors to be used in the Sherman–Morrison identity considering global coordinate system with additional computational cost. Also an additional inversion of a matrix, which has the size of the degrees of freedom (DOF) associated with the modified elements, is required. The iterative method was found to be effective only for small changes in a design, and the convergence rate might be slow or even divergent for large modifications in a design. Noor and Lowder[11] developed a modified iterative method, called the Taylor series–iterative technique (TS–IT). TS–IT improved the convergence rate and the accuracy of approximated solution and extended the applicable range of the reanalysis technique. Nevertheless, they concluded that the reduced basis technique appears to have the highest potential as a reanalysis technique. Kirsch[12] introduced the combined approximation method, combining binomial series expansion with the reduced basis method. However, these methods are valid only within certain move limits.

In this paper, a bound-free reanalysis technique, successive matrix inversion (SMI), is developed. SMI gives exact solutions for any variations to an initial FE system, that is, there is no restriction on the valid bounds of this method. The cost of reanalysis using SMI is flexible to the ratio of the changed portion to the initial stiffness matrix. For instance, when 10% of the stiffness matrix is changed, the computational effort using SMI is reduced by approximately 80%. Also, SMI is straightforward to understand and to implement.

First SMI is introduced in Sec. II and because SMI is not an approximation method, the computational cost of SMI is compared with conventional inverse techniques such as Cholesky decomposition, Gauss elimination, and QR decomposition in Sec. III. Three numerical examples are presented to demonstrate the efficiency and accuracy of SMI in Sec. IV. Finally, in Sec. V, some summary remarks are discussed.

This approach is used when modifying the existing structures where the local failures occur due to stress concentrations, buckling, etc. The size and material alterations are local, limited to small regions. Repair or replacement of existing structural members for battle damage and aging aircraft are some of the design applications. Another design application is the case where the finite difference method (FDM) is used for sensitivity information. SMI could be used to reduce the computational cost of FDM because the effect of a system response induced by a small modification of each design variable is calculated. Another area of application is reliability analysis in which only small regions are subjected to uncertainties of material properties, structural dimensions, geometric boundary conditions, and so on, the repetitive analyses could be performed by considering only those uncertain regions to obtain the system's responses with SMI efficiently and accurately. Also while quantifying the uncertainty, sampling methods typically are used where the modifications are local. SMI could be applied not only to static analysis but also to any analysis in which matrix inversion or decomposition is involved.

II. SMI Method

The main idea of the SMI method is to obtain the modified structural responses by considering only the modified portion of a structural stiffness matrix. Assume that the following initial simulation is performed using FEA:

$$[K_0]\{d_0\} = \{f\} \tag{1}$$

where $[K_0]$ is the initial stiffness matrix, $\{f\}$ is the force vector, and $\{d_0\}$ is the initial response vector. From this simulation, the inversion of stiffness matrix $[K_0]^{-1}$ and the initial response vector $\{d_0\}$ are known. Now, the stiffness matrix is changed with the stiffness modification matrix $[\Delta K]$,

$$([K_0] + [\Delta K])\{d\} = \{f\} \tag{2}$$

To evaluate the updated response vector $\{d\}$ for the changed global stiffness matrix $[K] = ([K_0] + [\Delta K])$, premultiplying Eq. (2) by $[K_0]^{-1}$ gives

$$([I] - [B])\{d\} = \{F\} \tag{3}$$

where

$$[B] = -[K_0]^{-1}[\Delta K] \tag{4}$$

$$\{F\} = [K_0]^{-1}\{f\} \tag{5}$$

In Eq. (5), $\{F\}$ is also the initial response vector $\{d_0\}$. Obviously, the updated response vector $\{d\}$ can be obtained by evaluating the inverse of the term in parentheses in Eq. (3). By use of a power series expansion, the inverse of the matrix $([I] - [B])^{-1}$ is given as follows:

$$([I] - [B])^{-1} = [I] + [B] + [B]^2 + [B]^3 + \cdots \tag{6}$$

This series expansion is known variously as the binomial series expansion, geometric series expansion, and Neumann series expansion. However, there are some limitations[13] for using this series expansion directly to find the inverse of the matrix $([I] - [B])^{-1}$:

1) A sufficient condition for the convergence of the series is the spectral radius of the matrix $[B]$ is less than unity.

2) The convergence could be quite slow in some cases.

Because of the first convergence limitation, there is a valid bound on the amount of design modification allowed for using the series method. Even though the convergence criterion is satisfied, using more than three series expansion terms for finding an inverse matrix might not be prudent from a computational cost point of view.

However, the inversion of the matrix $([I] - [B])^{-1}$ can be calculated from the element level of the infinite series expansion terms to alleviate the aforementioned problems. In Eq. (6), we define the matrix $[P]$ for the $[B]$ matrix series expansion terms as

$$[P] = [B] + [B]^2 + [B]^3 + \cdots \tag{7}$$

The elements of P can be obtained as follows:

$$P_{ij} = B_{ij}^{(1)} + B_{ij}^{(2)} + B_{ij}^{(3)} + \cdots + B_{ij}^{(k)} + \cdots \tag{8}$$

where $B_{ij}^{(k)}$ is the (i, j)th element of $[B]^k$ in Eq. (7). The kth recursive factor in the element series expansion $(r_{ij}^{(k)})$ in terms of Eq. (8) is obtained as

$$r_{ij}^{(k)} = B_{ij}^{(k+1)} \big/ B_{ij}^{(k)} \tag{9}$$

In the case where the recursive term is constant through all of the series expansion terms, that is, $r_{ij}^{(k)} = r_{ij}$, Eq. (8) can be expressed as follows:

$$P_{ij} = B_{ij}\left(1 + r_{ij} + r_{ij}^2 + r_{ij}^3 + r_{ij}^4 + \cdots\right) \tag{10}$$

The P_{ij} term can be obtained by assuming that there exists an original equation for the series expansion of each B matrix element as given in Eq. (10):

$$P_{ij} = B_{ij}/(1 - r_{ij}) \tag{11}$$

The right-hand-side term (the series expansion) of Eq. (10) is transformed into a simple expression as shown in Eq. (11). However, in the general case, it can be easily observed that the kth recursive term $r_{ij}^{(k)}$ is not same with the neighboring recursive terms, that is, the recursive term is not constant but variable for the series expansion. Hence, the transformation in Eq. (11) is not valid in general to obtain the series solution.

However, the variability of the series recursive term could be eliminated by decomposing the modified stiffness matrix into separate matrices as follows:

$$[\Delta K] = \sum_{j=1}^{N} \left[\Delta K^{(j)}\right] \tag{12}$$

where N is the total DOF in a structural model and $[\Delta K^{(j)}]$ is the matrix that has nonzero elements just in jth column and they represent $[\Delta K]$. When $[\Delta K^{(j)}]$ is considered in Eq. (4), the B matrix also has only jth column elements. Hence, when the series terms with the B matrix that has only one nonzero column vector is calculated, it is easily observed that the recursive terms for the B matrix are nothing but the (j, j)th element of the B matrix, as a constant value,

$$r = B_{jj} \tag{13}$$

Each element of $[P]$ is simply given as

$$P_{ij} = B_{ij}/(1 - r) \tag{14}$$

Then, the inverse of $[K^{(j)}] = ([K^{(j-1)}] + [\Delta K^{(j)}])$ is obtained as

$$\left[K^{(j)}\right]^{-1} = ([I] + [P])\left[K^{(j-1)}\right]^{-1} \tag{15}$$

where $[K^{(j-1)}]$ and $[K^{(j)}]$ are the modified stiffness matrices at $j-1$th and jth steps, respectively. In the next step, $[K^{(j-1)}]^{-1}$ is updated with the obtained $[K^{(j)}]^{-1}$ for calculating the next $[B^{(j)}]$ as given by Eq. (16), and here $[\Delta K^{(j)}]$ is the matrix for the next jth column. Hence, the successive iteration for obtaining the inverse of $[K] = ([K_0] + [\Delta K])$ could be summarized as

$$\left[B^{(j)}\right] = -\left[K^{(j-1)}\right]^{-1}\left[\Delta K^{(j)}\right] \tag{16}$$

$$r^{(j)} = B_{jj}^{(j)} \tag{17}$$

$$P_{ij}^{(j)} = B_{ij}^{(j)} \Big/ \left(1 - r^{(j)}\right) \tag{18}$$

$$\left[K^{(j)}\right]^{-1} = \left[K^{(j-1)}\right]^{-1} + \left[P^{(j)}\right]\left[K^{(j-1)}\right]^{-1} \tag{19}$$

where the initial $[K^{(0)}]^{-1}$ is given as $[K_0]^{-1}$ and the superscript (j) indicates the successive step. The number of successions is equal to the number of columns in $[\Delta K]$ that have nonzero elements. $[K^{(j-1)}]^{-1}$ denotes the successively corrected inversion of the modified stiffness matrix with $(j-1)$th column vector of $[\Delta K]$. The successively corrected inverse matrix could be obtained by any sequence of selecting $[\Delta K^{(j)}]$ because the contribution of each column vector of $[\Delta K]$ is independent. When $r^{(j)}$ is unity and Eq. (18) goes to infinity, it indicates the system is linearly dependent and the determinant is zero. Because $[\Delta K^{(j)}]$ is a column vector, the matrices $[B^{(j)}]$ and $[P^{(j)}]$ are also column vectors. Hence, the matrix operations in the iteration steps can be reduced to vector operations, and this procedure is shown in Fig. 1, where $\{Kb^{(j-1)}\}^T$ is the jth row vector of $[K^{(j-1)}]^{-1}$. Finally, after obtaining the inversion of the modified stiffness matrix, the system responses are obtained:

$$\{d\} = [K]^{-1}\{f\} \tag{20}$$

Again, the obtained solution is not an approximated solution but rather an exact one. In this work, SMI is developed by targeting the inverse of the changed global stiffness matrix $[K]^{-1}$. Note that, because the successive procedures in SMI have to be performed for each column vector, both symmetric and nonsymmetric stiffness matrices can be handled. Unlike Cholesky decomposition and Gauss elimination methods, SMI gives a stable solution because no pivoting is required. SMI can be applicable not only to a static analysis, but also to various analysis problems, including the eigenvalue problem. By the retransformation of the series expansion into an assumed original equation as given in Eq. (11), the requirement of the convergence criteria of series expansion is avoided and the computational effort is reduced significantly for a structure with a small ratio of the modified stiffness matrix.

III. Computational Cost Savings

The computational cost is compared with the conventional linear algebraic equation solvers by reckoning the number of the floating point operations (FLOPs), which represent the operation count. They include Cholesky decomposition, Gauss elimination, and QR decomposition. One FLOP is equal to the work required to compute one addition and one multiplication. For an $N \times N$ stiffness matrix, the numbers of FLOPs required to solve linear equations using different methods are shown in Table 1. It is known that Cholesky decomposition method is valid only for symmetric stiffness matrices and QR decomposition is more stable than Gauss elimination, which may require pivoting processes.

The earlier mentioned conventional techniques require another whole calculation for any modified $N \times N$ stiffness matrix ($[K] = [K_0] + [\Delta K]$) to obtain the new inverse stiffness matrix $[K]^{-1}$. However, in SMI, the computational cost is limited only to the modification matrix $[\Delta K]$. As already mentioned, the number of iterations needed in Fig. 1 is the same as the number of nonzero columns in $[\Delta K]$. Hence, the required number of FLOPs for obtaining the inverse of a changed stiffness matrix are $n(nN + N^2)$, where n is the number of nonzero element columns in $[\Delta K]$. The ratios of the computational cost of SMI to different conventional methods are shown in Fig. 2. The costs are calculated in terms of the required number of FLOPs. The SMI method can be applied to both symmetric and nonsymmetric stiffness cases as similar to Gauss elimination method. Hence, the computational cost of SMI and Gauss elimination method is compared. It is observed from Fig. 2 that there is no cost saving in calculating the inverse stiffness matrix with SMI for

Table 1 Number of FLOPs for using different linear equation methods

Method	FLOPs
Cholesky decomposition	$\frac{1}{3}N^3$
Gauss elimination	$\frac{2}{3}N^3$
QR decomposition	$2N^3$
SMI	$n(nN + N^2)$

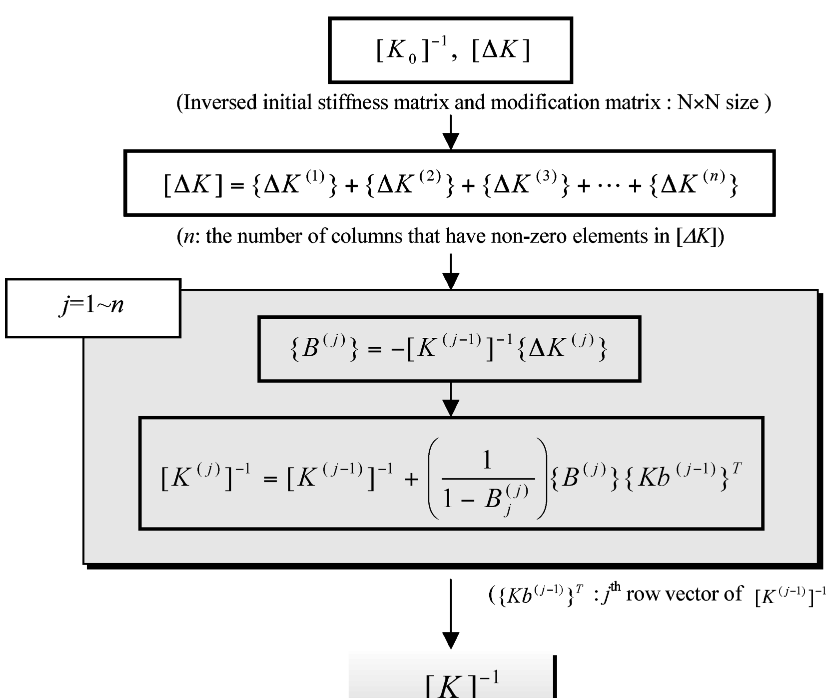

Fig. 1 SMI algorithm.

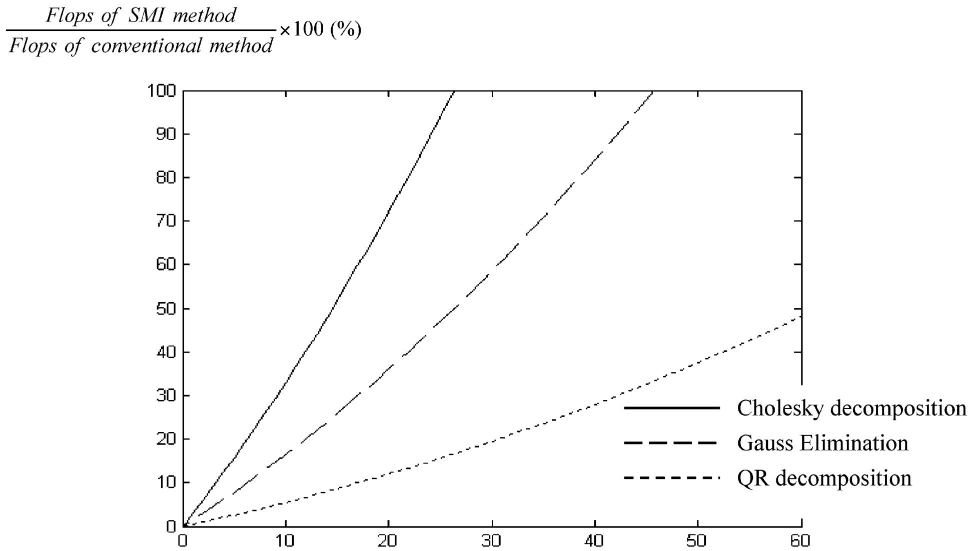

Fig. 2 Compared cost ratios of SMI to conventional methods; ratio of the changed column to total columns in a stiffness matrix: (n/N)100 (%).

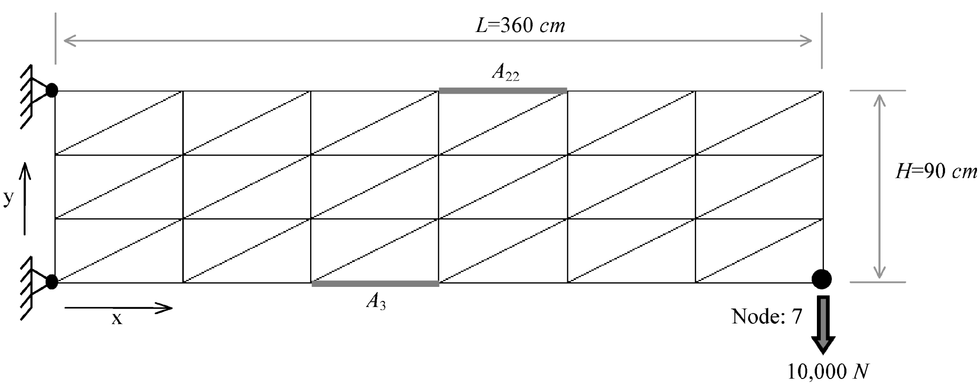

Fig. 3 Plane truss.

the ratio of the changed portion to the initial stiffness matrix that exceeds about 45%. However, for a 10% modification, cost saving is about 80%. SMI gives a stable and exact solution for any variation of the modification matrix, that is, the computational effort does not have any relation with the amount of modification, but rather it varies with the ratio of the changed portion to the whole stiffness matrix. Because SMI is employed on the matrix operation (inversion of stiffness matrix), any type of elements (rod, beam, plate, and so on) and also any geometric shapes could be handled as long as the number of DOF is not changed. Hence, this is a perfect tool for regional modifications of a target structure in design problems.

Note that the sparseness of the stiffness matrix has not been considered in the cost computation of each technique. Many engineering and scientific applications require the solution of sparse matrices. Most sparse matrix software take advantage of the sparseness to reduce the amount of storage and arithmetic operations required by keeping track of only the nonzero entries in the matrix. As shown in Fig. 1, only matrix–vector product and matrix–matrix addition are involved in the process of SMI for updating the initial inverse stiffness matrix. Especially, in Eq. (16), the vector ($[\Delta K^{(j)}]$) of the modification matrix typically has many zero elements. Also, the inverse stiffness matrix can be replaced by the decomposition matrices, which may have many zero elements. Therefore, the available sparse matrix operation techniques with certain storage schemes can be applied to the sparse matrix and vector multiplications in Eq. (16) directly to obtain benefits from the sparseness of the inverse and modification stiffness matrices.

IV. Numerical Examples

In this section, two examples with different types of elements, rod and plate, are presented to demonstrate the accuracy and efficiency of the SMI method.

Table 2 Results for plane truss (y-direction displacement at node 7)

α	A_1, cm^2	A_3, cm^2 [a]	Exact, cm	SMI, cm
0.00	0.5000	0.5000	−3.4020	−3.3980
0.04	0.4640	0.4592	−3.4248	−3.4207
0.08	0.4280	0.4184	−3.4514	−3.4473
0.12	0.3920	0.3776	−3.4830	−3.4788
0.16	0.3560	0.3368	−3.5209	−3.5166
0.20	0.3200	0.2960	−3.5674	−3.5631
0.24	0.2840	0.2552	−3.6258	−3.6213
0.28	0.2480	0.2144	−3.7012	−3.6965
0.32	0.2120	0.1736	−3.8024	−3.7975
0.36	0.1760	0.1328	−3.9453	−3.9402
0.40	0.1400	0.0920	−4.1625	−4.1568

[a]Initial A_3 and A_{22} are 0.5 and 0.5 cm^2.

A. Plane Truss

The plane truss is shown in Fig. 3 with elastic modulus $E = 206.0$ GPa and cross-section areas of all rod elements $A_0 = 0.5$ cm^2. The cross-section areas of the 3rd and 22th elements, A_3 and A_{22}, are selected as design variables. The modified cross-section areas are given by

$$\begin{pmatrix} A_3 \\ A_{22} \end{pmatrix} = \begin{pmatrix} A_0 \\ A_0 \end{pmatrix} + \alpha \begin{pmatrix} -0.0360 \\ -0.0408 \end{pmatrix} \tag{21}$$

The value of α is changed from 0 to 10, that is, the initial cross-section areas of A_3 and A_{22} (0.5 and 0.5) are modified up to 0.140 and 0.092, respectively. The y-direction displacements of node 7 for each α are shown in Table 2 and in Figure 4. The exact solutions are obtained by running simulations with GENESIS.[14] In Fig. 4, the exact displacements are shown as a nonlinear function of α. The

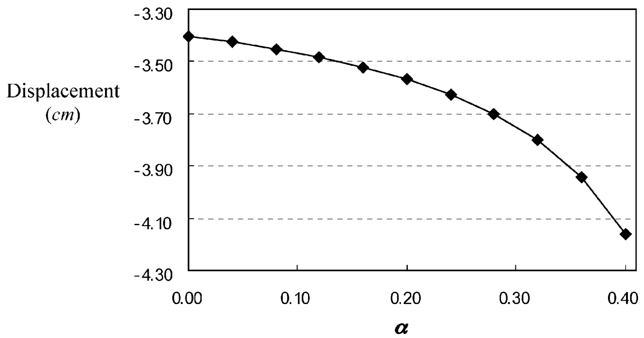

Fig. 4 Exact displacement (*y* direction) at node 7 for various values of α.

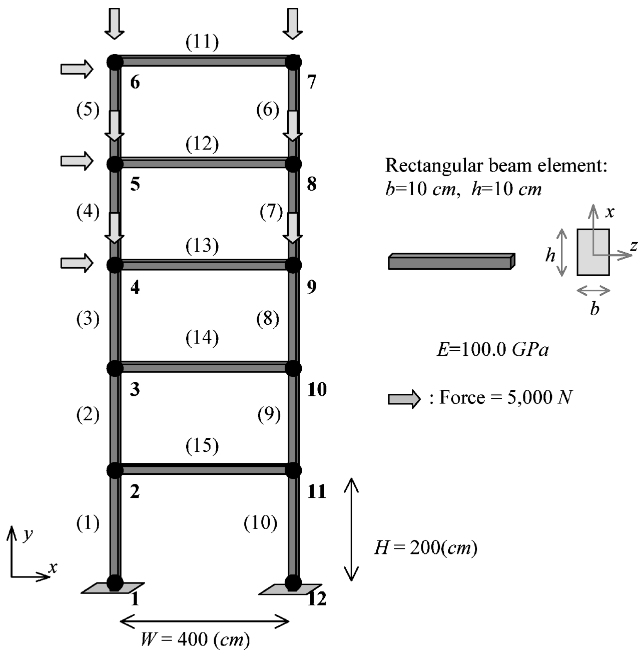

Fig. 5 Five-story building.

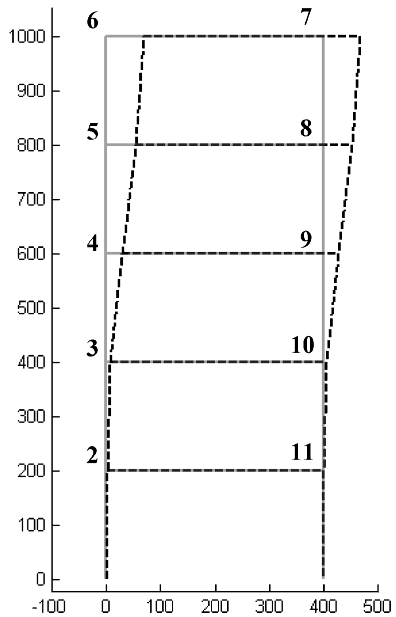

Fig. 6 Deformed shape.

ratio of the modified portion to the whole stiffness matrix is about 7.5%; hence, the computational cost of SMI is about 10% of the total cost of Gauss elimination method. The entire displacements are almost exact, and the trivial error (less than 0.1%) is induced from numerical round-off errors.

B. Five-Story Building

A five-story building subjected to point loads is modeled by using beam elements as shown in Fig. 5. The initial cross-sectional dimensions of all rectangular beam elements in the model are width $b = 10$ cm and height $h = 10$ cm. In this model, modified nodal displacements are investigated for the modifications of beam elements 1 and 2. The modifications are given to the cross-sectional

Table 3 Results for five-story building (*x*-direction displacement at node 6)

α	b_1	h_1	b_2	h_2	Exact, cm	SMI, cm
0.0	10.0	10.0	10.0	10.0	14.0629	14.0702
0.1	10.5	11.5	10.8	11.2	13.5095	13.4988
0.2	11.0	13.0	11.6	12.4	12.9880	12.9870
0.3	11.5	14.5	12.4	13.6	12.4728	12.4653
0.4	12.0	16.0	13.2	14.8	11.9555	11.9499
0.5	12.5	17.5	14.0	16.0	11.4375	11.4244
0.6	13.0	19.0	14.8	17.2	10.9257	10.9153
0.7	13.5	20.5	15.6	18.4	10.4292	10.4177
0.8	14.0	22.0	16.4	19.6	9.9569	9.9425
0.9	14.5	23.5	17.2	20.8	9.5164	9.5002
1.0	15.0	25.0	18.0	22.0	9.1125	9.0961
1.1	15.5	26.5	18.8	23.2	8.7476	8.7313
1.2	16.0	28.0	19.6	24.4	8.4220	8.4060
1.3	16.5	29.5	20.4	25.6	8.1342	8.1184
1.4	17.0	31.0	21.2	26.8	7.8817	7.8655
1.5	17.5	32.5	22.0	28.0	7.6614	7.6461
1.6	18.0	34.0	22.8	29.2	7.4698	7.4543
1.7	18.5	35.5	23.6	30.4	7.3035	7.2881
1.8	19.0	37.0	24.4	31.6	7.1594	7.1439
1.9	19.5	38.5	25.2	32.8	7.0346	7.0193
2.0	20.0	40.0	26.0	34.0	6.9263	6.9111

Table 4 Nodal displacement values for $\alpha = 2.0$

Displacement	Exact, cm	SMI, cm
$2x$	0.1604	0.1604
$2y$	−0.0001	−0.0001
$2\theta_z$	−0.0015	−0.0015
$3x$	0.5579	0.5579
$3y$	−0.0002	−0.0002
$3\theta_z$	−0.0023	−0.0023
$4x$	2.9232	2.922
$4y$	−0.0015	−0.0015
$4\theta_z$	−0.0114	−0.0114
$5x$	5.4072	5.4008
$5y$	−0.0028	−0.0029
$5\theta_z$	−0.0081	−0.008
$6x$	6.9263	6.9111
$6y$	−0.0037	−0.0038
$6\theta_z$	−0.0042	−0.0042
$7x$	6.9249	6.9098
$7y$	−0.0119	−0.0118
$7\theta_z$	−0.0041	−0.0041
$8x$	5.4056	5.3991
$8y$	−0.0114	−0.0113
$8\theta_z$	−0.0083	−0.0082
$9x$	2.9226	2.9214
$9y$	−0.0098	−0.0098
$9\theta_z$	−0.0105	−0.0105
$10x$	0.5625	0.5626
$10y$	−0.0069	−0.0069
$10\theta_z$	−0.0059	−0.0059
$11x$	0.1587	0.1587
$11y$	−0.0035	−0.0035
$11\theta_z$	−0.0004	−0.0004

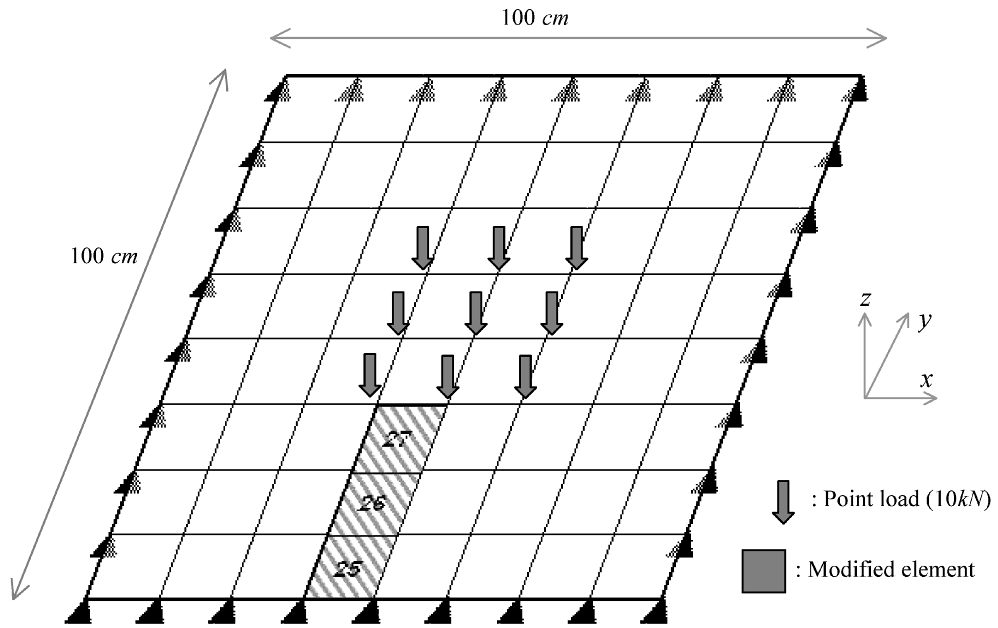

Fig. 7 Simply supported square plate (8 × 8 mesh), $E = 100.0$ GPa.

Table 5 Results for simply supported square plate

α	t_{25}	t_{26}	t_{27}	Exact, cm	SMI, cm
0.0	1.0	1.0	1.0	−8.7468	−8.7436
0.5	1.4	1.5	1.6	−8.3936	−8.3912
1.0	1.8	2.0	2.2	−8.1918	−8.1884
1.5	2.2	2.5	2.8	−8.0445	−8.0440
2.0	2.6	3.0	3.4	−7.9334	−7.9342
2.5	3.0	3.5	4.0	−7.8513	−7.8585
3.0	3.4	4.0	4.6	−7.7918	−7.7743
3.5	3.8	4.5	5.2	−7.7490	−7.7270
4.0	4.2	5.0	5.8	−7.7181	−7.7375
4.5	4.6	5.5	6.4	−7.6955	−7.7060
5.0	5.0	6.0	7.0	−7.6788	−7.7050

dimensions as follows:

$$\begin{Bmatrix} b_1 \\ h_1 \\ b_2 \\ h_2 \end{Bmatrix}_{\text{modified}} = \begin{Bmatrix} b_1 \\ h_1 \\ b_2 \\ h_2 \end{Bmatrix}_{\text{initial}} + \alpha \begin{Bmatrix} 5.0 \\ 15.0 \\ 8.0 \\ 12.0 \end{Bmatrix} \tag{22}$$

The value of α is changed from 0.1 to 2.0, that is, h_1 is changed up to four times to its initial value. Table 3 shows the displacement in the x direction at node 6 for various values of α. The exact solutions are obtained by running actual simulations for each value of α. On the other hand, SMI solutions are obtained by using with only 35% of the actual simulation computational effort. The deformed shape of the five-story building for $\alpha = 2$ from SMI is shown in Fig. 6 and the nodal displacement values in Table 4. The error between exact solutions and SMI solutions in Table 3 is induced only from the round-off error.

C. Simply Supported Square Plate

A square plate simply supported at the edges, with dimensions 100×100 cm, is investigated with the initial thicknesses 1.0 cm for all elements and the point loads as shown in Fig. 7. The thicknesses of the shaded elements (t_{25}, t_{26}, and t_{27}) in Fig. 7 are selected as design variables to give a nonsymmetrical modification, and they are modified with the following vector:

$$\{\Delta t\}^T = \{0.8 \quad 1.0 \quad 1.2\}$$

The changed design variables are obtained as

$$\{t^*\} = \{t_0\} + \alpha\{\Delta t\} \tag{23}$$

Table 5 shows the displacement in z direction at center node by performing complete analysis through GENESIS and the SMI method. The thickness is changed up to seven times of the initial thickness by varying the value of α from 0 to 5. SMI requires only about 20% of the complete analysis cost to obtain the modified solution. Because of round off errors, a maximum error of 0.3% is obtained between GENESIS and SMI solutions as shown in Table 5. However, note that SMI is implemented by coupling to the commercial FEM package, GENESIS, in this numerical examples. All of the information including stiffness matrices is extracted from GENESIS and stored in an output file with a specified precision (only six digits below the decimal point in an engineering data format) to be read by the main process of SMI. Hence, if the proposed method is coded in an FEM package with a usual data precision, such as, a double floating point precision, there should be no differences in solutions from the complete method and SMI. Moreover, the magnitude of the error does not necessarily increase as the target model becomes large.

V. Summary

In a modern structural design in which multidisciplinary analyses are involved, the need for efficient and accurate reanalysis technique is crucial because the modern structural design becomes more complex and larger. In this paper, a new reanalysis technique, the SMI method is introduced. It is shown that the computational effort to obtain exact solutions is significantly reduced compared to conventional methods with any variation for localized modifications. When general optimization problems or reliability problems are considered, a lot of research has been performed to reduce the design and random variables in a target system through sensitivity analysis. Hence, it is expected that the SMI method could reduce the overall computational cost in those problems that use sensitivity information. Although only static structural examples are shown for the demonstrations of efficiency and accuracy in this work, the SMI method could be applied to other structural analyses, such as an eigenvalue problem.

Acknowledgment

This project has been sponsored by the Air Force Office of Scientific Research through Grant F49620-00-1-0377.

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